

# A field theoretic approach to the energy momentum tensor for theories coupled with gravity

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We provide an field-theoretic algorithm of obtaining energy momentum tensor (EMT) for gravitationally coupled scalar field theories. The method is equally applicable to both minimal and non-minimal coupling. The algorithm illuminates the connection between the EMT, obtained by functional variation of the metric, and local balance of energy and momentum. It is of cardinal value for the proper identification of the EMT in the context of non-minimally coupled gravity theories.

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In the original formulation of general relativity (GR), Einstein introduced the physical energy momentum tensor (EMT) of the matter theory as the source of curvature [1] and curvature in turn determined the motion of the source. Subsequently, Hilbert enunciated an action principle to obtain Einstein's equations in the usual way, i.e. by extremizing an action. Various prescriptions for obtaining an EMT were developed [2–4]. The most popular algorithm [4] is to vary the action with respect to the background metric, leading to the definition

$$\Theta^{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_\Phi}{\delta g_{\mu\nu}} \quad (1)$$

where,  $S_\Phi$  is the generic action of the source fields minimally coupled to external gravity. The EMT thus obtained is symmetric and covariantly conserved. Physics of interactions other than gravity are described by field theories in the flat (Minkowskian) tangent space. In this context it may be noted that Noether proved [5] that corresponding to a continuous symmetry of the field theoretic action there exists a conserved current. Specifically, translational symmetry leads to a conserved EMT [6]. This approach has, however, been proved to be difficult in curved space-time and therefore, in the theories of gravitation the Noether's algorithm retired in the background. However, one should remember that the EMT obtained from equation (1) has no obvious connection [7] with the physical definition of EMT that involves flow of energy and momentum [6]. In the EMT obtained by Noether's method this connection is strong because here the conservation of EMT is connected with the translational symmetry of the system and the energy momentum four-vector is generator of translation. Actually our faith on the definition (1) relies on examples where the two methods give the same result. But all the examples where this equivalence is demonstrated are minimally coupled theories. It will be indeed interesting to investigate the connection in the context of non-minimal coupling.

For non-minimally coupled theories the definition (1) can not be applied as such. The usual way out is to write the equation of motion corresponding to the metric  $g_{\mu\nu}$  and read-off the EMT by rearranging different terms therein. Naturally there is ambiguity as the rearrangement follows no definitive prescription [8–17]. This ambiguity may only be resolved if a dynamical approach is available to identify the source producing curvature. In this paper, we present a field-theory based algorithm (applicable to both minimally and non-minimally coupled theories) to this end.

The essence of our approach consists of the following. We follow the spirit inherent in (1) which is the response of the source due to variation of the background metric where the metric is external and not dynamical. As is well-known the primitive definition of EMT is based on the balance of energy and momentum which is a local phenomena. Hence, to construct an EMT from the physical perspective the locally inertial coordinates are particularly suitable. In these coordinates, the metric assumes value as in the flat (Minkowski) space time and its first derivatives vanish. Nevertheless the curvature of the background spacetime shows up in the nonvanishing second derivatives of the metric. The crux of our method is to view the corresponding action as an auxiliary field theory in the tangent space at P where the curved space and the tangent space has an infinitesimal overlap. The whole process can be summarised in the following algorithm:

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1. From the original theory subtract the dynamic part for pure gravity ( i.e. Einstein–Hilbert action). This will give the scalar field theory interacting with external gravity.
2. Express the resulting action in the locally inertial coordinates, at a point P, which have the properties

$$g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \quad (2)$$

$$\partial_\lambda g_{\mu\nu} = 0 \quad (3)$$

$$\partial_\lambda \partial_\rho g_{\mu\nu} \neq 0 \quad (4)$$

3. The action obtained in step(2) will now be considered as a field theory in the Minkowskian tangent space at the point P, with metric  $\eta_{\alpha\beta}$ .  $\phi$  and  $g_{\mu\nu}$  are now respectively scalar and second rank tensor fields in this tangent space. Obviously, only second derivatives of  $g_{\mu\nu}$  will appear in this theory. Note that the dynamics of this new field theory is *not* constrained by equations (2, 3, 4).
4. Now compute the EMT by applying Noether's theorem using the translational symmetry of the new theory. Since the energy-momentum four vector is a generator of translational symmetry the EMT actually corresponds to the balance of energy and momentum all over the tangent space including point P.
5. From steps (1), (2) and (3) it can be understood that the EMT obtained in step (4) is same as the EMT of the scalar field theory interacting with external gravity of step (1) locally at a point P in the region of overlap.
6. The final task is to express the EMT in step(5) in terms of general coordinates in curved space-time. Care should be taken in this step so that all the terms that appear in this EMT has unambiguous geometric meaning. What we mean by the phrase 'unambiguous geometric meaning' is clarified in the following, see below equation(18).
7. The EMT thus obtained should serve as the source for the gravitational field in the original theory.

From the above description of the proposed method it is apparent that the procedure is applicable for a generic scalar field theory coupled to gravity. For definiteness we take a non-minimally coupled quintessence model to illustrate our method though the same algorithm is applicable in principle for different dynamics of the scalar field, such as k-essence.

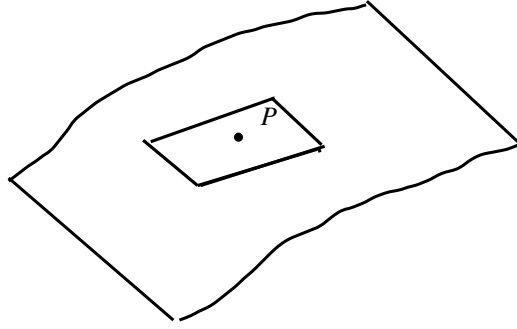


FIG. 1: Field theory defined on the tangent space

We start with the following action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{pl}^2}{2} (1 - F(\phi)) R - \frac{1}{2} g^{\mu\nu} (\nabla_\mu \phi) (\nabla_\nu \phi) - V(\phi) \right] \quad (5)$$

Note that a non-zero  $F(\phi)$  signifies nonminimal coupling. In (5),  $M_{pl}^2$  is given by  $(8\pi G)^{-1}$ .

As the first step of our procedure we abstract from (5) the form of the theory coupled with curved space-time when the metric is external. This leads to the Lagrangian

$$\mathcal{L}_\phi = -\sqrt{-g} \left[ \frac{M_{pl}^2}{2} F(\phi) R + \frac{1}{2} g^{\mu\nu} (\nabla_\mu \phi) (\nabla_\nu \phi) + V(\phi) \right] \quad (6)$$

Note that metric in (6) is a background field and influences the motion through the coupling with the scalar field through curvature.

So far, the coordinate system was general, charting the curved spacetime. The next step of our method is to concentrate at the neighbourhood of the point  $P$  (see fig.1) and adopt the locally inertial coordinates. The Riemann tensor is expressed, using (2), (3), in the form [7],

$$R_{\alpha\beta\gamma\delta} = \frac{1}{2} (g_{\beta\gamma,\alpha\delta} - g_{\alpha\gamma,\beta\delta} - g_{\beta\delta,\alpha\gamma} + g_{\alpha\delta,\beta\gamma}) \quad (7)$$

The Ricci tensor is

$$R_{\beta\delta} = \eta^{\alpha\gamma} R_{\alpha\beta\gamma\delta} \quad (8)$$

The Ricci scalar is obtained on another contraction as,

$$R = \eta^{\alpha\gamma} \eta^{\beta\delta} R_{\alpha\beta\gamma\delta} = - (\eta^{\alpha\gamma} \partial_\lambda \partial^\lambda g_{\alpha\beta} - \partial^\alpha \partial^\beta g_{\alpha\beta}) \quad (9)$$

The Lagrangian (6) can now be written in terms of the locally inertial coordinates, using (9) as

$$\mathcal{L} = \frac{M_{pl}^2}{2} F(\phi) \{ \eta^{\alpha\gamma} \partial_\lambda \partial^\lambda g_{\alpha\beta} - \partial^\alpha \partial^\beta g_{\alpha\beta} \} - \frac{1}{2} \eta^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \quad (10)$$

Note that due to the condition (2),  $\sqrt{-g} = 1$ . The region in which the locally inertial coordinates are defined is an infinitesimal patch containing the point  $P$  in the curved space-time. The tangent space at  $P$  contains this infinitesimal patch as well. In this flat (Minkowski) tangent space, we can choose a coordinate system that becomes identical with the locally inertial frame in the region of overlap.

As the next step of our algorithm the Lagrangian (10) is viewed as a new field theory in the flat (Minkowski) spacetime with metric  $\eta_{\mu\nu}$ , where two different kinds of fields are involved. One is the scalar field  $\phi$  and the other is  $g_{\mu\nu}$ , a second rank tensor field. In this field theory (10) these fields together form a closed system. The theory has translational symmetry (as a part of the more general Poincare symmetry).

We are now in a position to apply Noether theorem to get the conserved EMT of (10). Before commencing the dynamical analysis of the theory note that the model (10) in this new avatar is a theory in the tangent space and is not subject to the restrictions, (2) and (3) which have played their role in expressing the Lagrangian (6) in terms of the local inertial coordinates and have nothing to do with the Lagrangian (10).

The equation of motion for the  $\phi$  field is obtained as usual,

$$\square\phi - \frac{M_{pl}^2}{2} F' R - V' = 0 \quad (11)$$

In the above,  $A'$  denotes differentiation of  $A(\phi)$  with respect to  $\phi$ .

A characteristic feature in the Lagrangian (10) is the presence of second derivatives of the tensor field  $g_{\mu\nu}$ . Thus it is a higher derivative field theory [18]. The equation of motion for the  $g_{\mu\nu}$  field is [19],

$$\partial_\mu \partial_\nu \left[ \frac{\partial \mathcal{L}}{\partial (\partial_\mu \partial_\nu g_{\alpha\beta})} \right] - \partial_\mu \left[ \frac{\partial \mathcal{L}}{\partial (\partial_\mu g_{\alpha\beta})} \right] + \frac{\partial \mathcal{L}}{\partial g_{\alpha\beta}} = 0 \quad (12)$$

Using (10), we get the explicit form,

$$\partial_\mu \partial_\nu \left[ \frac{M_{pl}^2}{2} F(\phi) \right] \{ \eta^{\alpha\nu} \eta^{\mu\beta} - \eta^{\mu\nu} \eta^{\alpha\beta} \} = 0 \quad (13)$$

Apparently this equation makes no reference to the field  $g_{\mu\nu}$ . However, eliminating  $F(\phi)$  from equation(13) and (11), the explicit equation of motion for the fields  $g_{\mu\nu}$  can be obtained. Also one can alternatively write equation (13) as,

$$\partial^\alpha \partial^\beta F - \eta^{\alpha\beta} \square F = 0 \quad (14)$$

Operating by  $\partial_\alpha$  on both sides we find that the left hand side vanishes.  $F(\phi)$  is an implicate function of  $x$  and the Green's function for (14) is not obtainable[35]. It physically means that the present analysis does not depend on any specific choice of  $F(\phi)$ , justifying our claim that the algorithm is valid for a generic non-minimal coupling of the type (5).

The construction of the EMT,  $T^{\mu\nu}$  will now be detailed. Following Noether's prescription,

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} (\partial^\nu \phi) - \eta^{\mu\nu} \mathcal{L} + \frac{\partial \mathcal{L}}{\partial(\partial_\lambda \partial_\mu g_{\alpha\beta})} (\partial_\lambda \partial^\nu g_{\alpha\beta}) - \partial_\lambda \left( \frac{\partial \mathcal{L}}{\partial \partial_\lambda \partial_\mu g_{\alpha\beta}} \right) (\partial^\nu g_{\alpha\beta}) \quad (15)$$

which gives

$$\begin{aligned} T^{\mu\nu} = & -(\partial^\mu \phi)(\partial^\nu \phi) + \eta^{\mu\nu} \left[ \frac{M_{\text{pl}}^2}{2} F(\phi) R + \frac{1}{2} \eta^{\alpha\beta} (\partial_\alpha \phi)(\partial_\beta \phi) + V(\phi) \right] \\ & - \left[ \frac{M_{\text{pl}}^2}{2} (\eta^{\lambda\mu} \eta^{\sigma\beta} - \eta^{\sigma\mu} \eta^{\lambda\beta}) \partial^\nu \partial_\sigma g_{\lambda\beta} \right] F(\phi) \\ & + \partial_\sigma \left[ \frac{M_{\text{pl}}^2}{2} (\eta^{\lambda\mu} \eta^{\sigma\beta} - \eta^{\sigma\mu} \eta^{\lambda\beta}) F(\phi) \right] \partial^\nu g_{\lambda\beta} \end{aligned} \quad (16)$$

An explicit check of the conservation of  $T^{\mu\nu}$  is due. A straight forward calculation shows,

$$\partial_\mu T^{\mu\nu} = 0 \quad (17)$$

In arriving at the above conservation law we have used the equations of motion (11) and (13). Now that we have obtained a conserved EMT for the field theory (10) it is required to be identified with the EMT of (5) in locally inertial coordinates at  $P$ . In that case the status of  $g_{\mu\nu}$  field will be restored as the metric. The restrictions (2) and (3) will consequently become operative. Naturally the form (16) in the flat space is not suitable for this identification as the last term contains first derivatives of  $g_{\mu\nu}$  which will vanish and therefore the conservation (17) will be disturbed. Such situations must be avoided before we identify our EMT in flat space at  $P$  with the EMT of (5) in local inertial coordinates. Hence rewriting the expression (16) for  $T^{\mu\nu}$  as

$$\begin{aligned} T^{\mu\nu} = & -(\partial^\mu \phi)(\partial^\nu \phi) + \eta^{\mu\nu} \left[ \frac{M_{\text{pl}}^2}{2} F(\phi) R + \frac{1}{2} \eta^{\alpha\beta} (\partial_\alpha \phi)(\partial_\beta \phi) + V(\phi) \right] \\ & - \frac{M_{\text{pl}}^2}{2} (\eta^{\lambda\mu} \eta^{\sigma\beta} - \eta^{\sigma\mu} \eta^{\lambda\beta}) (\partial^\nu \partial_\sigma g_{\lambda\beta}) F(\phi) \\ & + \frac{M_{\text{pl}}^2}{2} (\eta^{\lambda\mu} \eta^{\sigma\beta} - \eta^{\sigma\mu} \eta^{\lambda\beta}) \partial^\nu \{g_{\lambda\beta} \partial_\sigma F(\phi)\} - \frac{M_{\text{pl}}^2}{2} [\eta^{\lambda\mu} (\partial^\beta \partial^\nu F) g_{\lambda\beta} - \eta^{\lambda\beta} (\partial^\mu \partial^\nu F) g_{\lambda\beta}] \end{aligned} \quad (18)$$

we obtain a form of EMT in the flat space that is devoid of any explicit occurrence of first derivatives of  $g_{\mu\nu}$ . It should be noted that (18) still refers to the flat space field theory and  $g_{\mu\nu}$  is still a second rank tensor field. Also, before we can import the EMT (18) to the local patch at  $P$  and express it in terms of general coordinates, we have to ensure that all the terms have unambiguous correspondence with geometric objects. For example, the first and second term of (18) are already in a form that has such a correspondence, but not the third term, owing to the presence of the factor  $(\partial^\nu \partial_\sigma g_{\lambda\beta})$ . We have to improve the EMT (18) so as to get rid of such ambiguity.

A possible way is to add a term  $\partial_\sigma M^{\sigma\mu\nu}$  to the EMT (18), where  $M^{\sigma\mu\nu}$  is a third rank tensor antisymmetric in its first two indices such that  $\partial_\mu \partial_\sigma M^{\sigma\mu\nu}$  vanishes identically. This ensures the conservation of the improved EMT [20]. An appropriate choice is

$$M^{\sigma\mu\nu} = \eta^{\nu\beta} (\eta^{\mu\lambda} \eta^{\rho\sigma} - \eta^{\mu\rho} \eta^{\lambda\sigma}) [(\partial_\rho g_{\lambda\beta}) F(\phi) - 3 \{ \partial_\rho F(\phi) \} g_{\lambda\beta}] \quad (19)$$

Note that  $M^{\sigma\mu\nu}$  has the required antisymmetric in  $\sigma$  and  $\mu$ . Adding  $\frac{M_{\text{pl}}^2}{2} \partial_\sigma M^{\sigma\mu\nu}$  with  $T^{\mu\nu}$  in (18), we get the improved tensor,

$$\begin{aligned} \Theta^{\mu\nu} = & -(\partial^\mu \phi)(\partial^\nu \phi) + \eta^{\mu\nu} \left[ \frac{M_{\text{pl}}^2}{2} F(\phi) R + \frac{1}{2} \eta^{\alpha\beta} (\partial_\alpha \phi)(\partial_\beta \phi) + V(\phi) \right] - M_{\text{pl}}^2 R^{\mu\nu} F(\phi) \\ & + \frac{M_{\text{pl}}^2}{2} \eta^{\nu\rho} (\eta^{\lambda\mu} \eta^{\sigma\beta} - \eta^{\sigma\mu} \eta^{\lambda\beta}) \partial_\rho \{g_{\lambda\beta} \partial_\sigma F(\phi)\} - \frac{M_{\text{pl}}^2}{2} [\eta^{\lambda\mu} (\partial^\beta \partial^\nu F) g_{\lambda\beta} - \eta^{\lambda\beta} (\partial^\mu \partial^\nu F) g_{\lambda\beta}] \\ & + \frac{M_{\text{pl}}^2}{2} \eta^{\nu\beta} (\eta^{\lambda\mu} \eta^{\rho\sigma} - \eta^{\rho\mu} \eta^{\lambda\sigma}) [\partial_\rho \{g_{\lambda\beta} \partial_\sigma F(\phi)\} - 3 \partial_\sigma \{g_{\lambda\beta} \partial_\rho F(\phi)\}] \end{aligned} \quad (20)$$

Here we have used the relation

$$\begin{aligned}
(\eta^{\lambda\mu}\eta^{\sigma\beta} - \eta^{\sigma\mu}\eta^{\lambda\beta}) \{\partial^\nu \partial_\sigma g_{\lambda\beta}\} F(\phi) &= 2R^{\mu\nu} F(\phi) + (\eta^{\mu\beta}\eta^{\nu\delta}\Box g_{\beta\delta} - \eta^{\nu\delta}\partial^\lambda \partial_\mu g_{\lambda\beta}) F(\phi) \\
&= 2R^{\mu\nu} F(\phi) + \partial_\sigma [\eta^{\nu\beta} (\eta^{\mu\lambda}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\lambda\sigma}) \partial_\rho g_{\lambda\beta} F] \\
&\quad - \eta^{\nu\beta} (\eta^{\mu\lambda}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\lambda\sigma}) \partial_\rho (g_{\lambda\beta} \partial_\sigma F)
\end{aligned} \tag{21}$$

obtained using equation (7) and (13).

The final step of our algorithm is to identify the EMT (20) to that of the original theory (5) in the local inertial coordinates at the point P and then convert it to the general coordinates. Once this identification is done,  $g_{\mu\nu}$  becomes the metric. The different terms on the right hand side of (20) are in appropriate tensorial form suitable to be lifted to general coordinates. For instance,

$$\frac{M_{\text{pl}}^2}{2} \eta^{\nu\beta} (\eta^{\lambda\mu}\eta^{\rho\sigma} - \eta^{\rho\mu}\eta^{\lambda\sigma}) \partial_\rho (g_{\lambda\beta} \partial_\sigma F(\phi)) \rightarrow \frac{M_{\text{pl}}^2}{2} g^{\nu\beta} (g^{\lambda\mu} g^{\rho\sigma} - g^{\rho\mu} g^{\lambda\sigma}) \nabla_\rho (g_{\lambda\beta} \nabla_\sigma F(\phi)). \tag{22}$$

The right hand side of (22) is in covariant form and passes to the left hand side when the defining properties (2), (3) and (4) are used. Similar generalization of the other terms is obvious, considering that  $F$  is a scalar. One can now readily obtain the expression for the EMT in general coordinates,

$$\Theta^{\mu\nu} = -(\nabla^\mu \phi)(\nabla^\nu \phi) + g^{\mu\nu} \left[ \frac{1}{2} g^{\alpha\beta} (\nabla_\alpha \phi)(\nabla_\beta \phi) + V(\phi) \right] - M_{\text{pl}}^2 F(\phi) G^{\mu\nu} - M_{\text{pl}}^2 g^{\mu\nu} \Delta F + M_{\text{pl}}^2 \nabla^\mu \nabla^\nu F \tag{23}$$

where  $\Delta F = \nabla_\mu \nabla^\mu F(\phi)$  and  $G^{\mu\nu}$  is the Einstein tensor. Note that metric compatibility is assumed.

As can be checked directly, our algorithm, based on Noether theorem and dynamics of fields, leads to an EMT which is symmetric and covariantly conserved. Based as it is on the local conservation of energy-momentum, this EMT serves as the source of gravity *a la* Einstein. Hence following the original spirit of general relativity, we write the equation of motion of non-minimally coupled quintessence model as

$$G^{\mu\nu} = -\frac{1}{M_{\text{pl}}^2} \Theta^{\mu\nu} \tag{24}$$

The correspondence with the Hilbert action principle must be investigated. For the minimal coupling  $F(\phi) = 0$  and the equivalence is obvious. Much more interesting and really nontrivial is the case when  $F(\phi) \neq 0$ , i.e., non-minimally coupled theories. Below we will show that our algorithm produces consistent result for such theories as well.

Non-minimally coupled theories appear in different contexts, e.g., quantum corrections [21], renormalization of classical theory [22], in the string theoretic context [23] and in the Scalar-tensor theories [24, 25]. Now a days such theories are intensely investigated in context of dark energy [26–34]. However, in the literature there is certain ambiguity [8–17] in identifying the EMT for a non-minimal theory. To illustrate this, let us first write the equation of motion obtained by varying the metric of (5) (with  $F(\phi) \neq 0$ ),

$$M_{\text{pl}}^2 (1 - F) G_{\mu\nu} = (\nabla_\mu \phi \nabla_\nu \phi) - g_{\mu\nu} \left[ \frac{1}{2} g^{\alpha\beta} (\nabla_\alpha \phi)(\nabla_\beta \phi) + V(\phi) \right] + M_{\text{pl}}^2 g_{\mu\nu} \Delta F - M_{\text{pl}}^2 \nabla_\mu \nabla_\nu F = \tilde{\Theta}_{\mu\nu} \tag{25}$$

One can keep the equation as it is [14] and interpret  $\tilde{\Theta}_{\mu\nu}$  as the EMT, which is clearly not covariantly conserved, thereby emphasising the deviation from the Einstein structure (24). This non-conservation has been criticised in [10]. Alternatively, conserved EMT can be identified from (25) by algebraic manipulations which may take different courses [9, 10, 12, 14, 15] leading to EMTs with apparently different forms, e.g.

$$\Theta^D_{\mu\nu} = \frac{\tilde{\Theta}_{\mu\nu}}{(1 - F)} \tag{26}$$

$$\Theta^A_{\mu\nu} = \tilde{\Theta}_{\mu\nu} + M_{\text{pl}}^2 F G_{\mu\nu} \tag{27}$$

Looking back at our equation (23) it is easy to see that it agrees with (27) which is the form obtained in [9, 10]. However, the algebraic manipulation which led to their result has no apparent connection with the conservation of energy and momentum. This agreement asserts that, like the standard general relativity, the dynamically constructed EMT serves as the source even for non-minimally coupled gravity theories. In fact, it does something more. It shows that only the covariantly conserved EMT has dynamical basis and should be identified as source.

In this paper we have developed a novel algorithm to obtain the Energy momentum tensor (EMT) of scalar field theories coupled with gravity. The method rests on the corresponding theory where gravity is non dynamical. Locally

inertial coordinates are adopted where the first derivative of the metric vanishes in an infinitesimal patch. We have then defined a field theory in the tangent space by the same action as the one obtained in the locally inertial coordinates. This theory has translation symmetry and the conserved Noether current gives an EMT. Identifying it as the EMT of the original theory in the locally inertial coordinate, we generalized the same to curved coordinates.

When the coupling of the scalar field with gravity is minimal, the standard form of the EMT is reproduced from the expression (23) derived here. More interesting area of application is in context of the non minimally coupled scalar field theories, where different prescriptions [8–17] are advocated, since no systematic method exists to write the EMT. The dynamical algorithm proposed here is able to provide a particular form of the EMT which allows one to write the equations of motion (24) in the same way as the minimally coupled theories. It is expected that the methodology adopted here will have wider applicability, e.g., in more general scalar-tensor theories.

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